



DOI: 10.29026/oea.2018.180012

# An accurate design of graphene oxide ultrathin flat lens based on Rayleigh-Sommerfeld theory

Guiyuan Cao, Xiaosong Gan, Han Lin and Baohua Jia\*

Centre for Micro-Photonics, Faculty of Engineering, Science and Technology, Swinburne University of Technology. John Street, Hawthorn, VIC 3122, Australia

\* Correspondence: B H Jia, Email: [bjia@swin.edu.au](mailto:bjia@swin.edu.au)

## **This file includes:**

Section 1: Lens design method

Section 2: Experimental setup of the laser fabrication system (Fig. S1)

Section 3: Experimental setup of the GO lens characterization (Fig. S2)

Supplementary information is available for this paper at <https://doi.org/10.29026/oea.2018.180012>

Section 1: Lens design method

The field distribution in the focal region of the ultrathin lens can be calculated using the Rayleigh-Sommerfeld (RS) diffraction theory<sup>1</sup>:

$$U_2(r_2, \theta_2, z) = -\frac{i}{\lambda} \iint U_1'(r_1, \theta_1) \frac{\exp(ikr)}{r} \cos(\mathbf{n}, \mathbf{r}) dr_1 d\theta_1, \quad (1)$$

where  $r = [z^2 + (x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2} = [z^2 + r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)]^{1/2}$ ,  $\mathbf{n}$  denotes the unit vector normal toward the observe plane,  $\mathbf{r}$  is the unit vector of  $r$  direction from  $r_1$  to  $r_2$  as shown in Fig. 1(a). Therefore, we can obtain the field distribution in the focal region in cylindrical coordinate system:

$$U_2(r_2, \theta_2, z) = -\frac{i}{\lambda} \int_0^{2\pi} \int_0^\infty U_1'(r_1, \theta_1) \frac{\exp(-ik\sqrt{z^2 + r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)})}{z^2 + r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)} z r_1 dr_1 d\theta_1. \quad (2)$$

To design the GO lens with the targeted focal length  $f$  and diameter  $D$ , we only consider the intensity distribution on the  $z$  axis, namely  $r_2=0, z=f$ . Therefore, the field distribution along  $z$  axis is:

$$U_2(f) = -\frac{i}{\lambda} \int_0^{2\pi} \int_0^\infty U_1'(r_1, \theta_1) \frac{\exp(-ik\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} z r_1 dr_1 d\theta_1 = -\frac{i2\pi}{\lambda} \int_0^\infty U_1'(r_1) \frac{\exp(-ik\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} f r_1 dr_1. \quad (3)$$

Now, for the targeted focal length  $f$ ,  $U_2(f)$  is decided by  $r_1$  only. Based on the Euler's equation, the field distribution along  $z$  axis can be rewritten as:

$$U_2(r_1) = \frac{-2\pi z}{\lambda} \left[ i \int_0^\infty U_1(r_1) \frac{\cos(-k\sqrt{z^2 + r_1^2})}{z^2 + r_1^2} r_1 dr_1 - \int_0^\infty U_1(r_1) \frac{\sin(-k\sqrt{z^2 + r_1^2})}{z^2 + r_1^2} r_1 dr_1 \right]. \quad (4)$$

Therefore, the intensity distribution on the  $z$  axis can be simplified to:

$$I(r_1) = \text{abs}([U_2(f)]^2) = \left( \frac{2\pi f}{\lambda} \right)^2 \left[ \left( \int_0^\infty U_1(r_1) \frac{\cos(-k\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} r_1 dr_1 \right)^2 + \left( \int_0^\infty U_1(r_1) \frac{\sin(-k\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} r_1 dr_1 \right)^2 \right]. \quad (5)$$

To find out the maximal destructive interference positions on the intensity distribution  $I(r_1)$ , taking the derivative of equation (5), we can obtain the contribution of  $I(r_1)$  on point  $f$  along  $r_1$ :

$$\frac{dI}{dr_1} = 2 \times \left( \frac{2\pi f}{\lambda} \right)^2 \left[ \left( \frac{\cos(-k\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} r_1 \int_0^\infty \frac{\cos(-k\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} r_1 dr_1 \right) + \left( \frac{\sin(-k\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} r_1 \int_0^\infty \frac{\sin(-k\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} r_1 dr_1 \right) \right]. \quad (6)$$

However,

$$\int_0^\infty \frac{\cos(-k\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} r_1 dr_1 = \int_0^\infty \frac{\cos(-k\sqrt{f^2 + r_1^2})}{f^2 + r_1^2} \frac{1}{2} d(r_1^2 + f^2) = \int_0^\infty \frac{\cos(-kR)}{R^2} dR^2 = \int_0^\infty \frac{\cos(-kR)}{R} dR. \quad (7)$$

where  $R = (f^2 + r_1^2)^{1/2}$ .

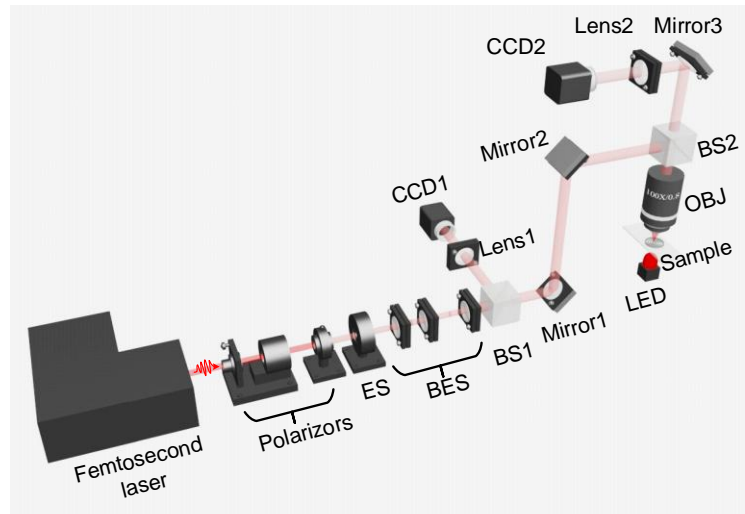
As we know:

$$\cos R = 1 - \frac{R^2}{2!} + \frac{R^4}{4!} + (-1)^n \frac{R^{2n}}{(2n)!}, \quad (8)$$

$$\sin R = R - \frac{R^3}{3!} + \frac{R^5}{5!} + (-1)^n \frac{R^{2n+1}}{(2n+1)!}, \quad (9)$$

where  $n$  is integer greater than or equal to 0. There is no analytic expression of equations (8) and (9), therefore there is no analytic expression of indefinite integral equation (7). We have to use Matlab to find out the maximal destructive interference positions. When we use Matlab, we can program from equation (5) directly.

## Section 2: Experimental setup of the laser fabrication system



**Fig. S1** | Experimental setup of the laser fabrication system. ES: electronic shutter; BES: beam expanding system; BS1 and BS2: beam splitter; LED: light-emitting diode; Sample: GO film; OBJ: objective; CCD1 and CCD2: charge coupled device

## Section 3: Experimental setup of the GO lens characterization



**Fig. S2** | Experimental setup of the GO lens characterization.

## Supplementary information references

1. Gu M. *Advanced Optical Imaging Theory* (Springer, 2000).